QUASIOPTIMAL CONVERGENT ADAPTIVE FEM FOR AN INDEFINITE ELLIPTIC PDE

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Adaptive finite element methods (AFEM) employ automatic local mesh refinement based on reliable and efficient a posteriori error bounds. These methods provide accurate and optimally convergent numerical solutions even when the solution does not have full regularity. In this talk, we discuss the convergence and quasi-optimal rate of convergence of an adaptive finite element method(AFEM) for a general second-order non-selfadjoint elliptic PDE:

$$\begin{aligned} -\nabla \cdot (A\nabla u) + b \cdot \nabla u + cu &= f \text{ in } \Omega \\ u &= 0 \text{ on } \partial\Omega \end{aligned}$$

with convection term $b \in [L^\infty(\Omega)]^d$ and using minimal regularity of the dual problem i.e. the solution of the dual problem has only H^1 regularity. The PDE is assumed to satisfy a Garding type inequality and thereby requires subtle arguments when the convective term is only an L^∞ function. The results derived here extends the result of J. M. Cascon, C. Kreuzer, R. H. Nochetto and K. G. Siebert, *SIAM J. Numer. Anal.*, 46:2524-2550, 2008. The theoretical results are illustrated by numerical experiments.

PRIMAL HYBRID METHOD FOR GRADIENT TYPE NON-LINEARITY PARABOLIC PROBLEMS

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This article develops the primal hybrid finite element method with Lagrange multipliers to approximate nonlinear parabolic initial-boundary value problems with gradient type non-linearity. A modified elliptic projection is used to produce optimal order error estimates for the semi-discrete and backward Euler-based complete discrete schemes. In addition, error estimates in L^{∞}-norm are established which are optimal in nature. Superconvergence result of the gradient in L^{∞}-norm is discussed for the error between the primal hybrid solution and elliptic projection. As a bi-product, the proposed analysis provides optimal error analysis for non-conforming CR-elements. Finally, numerical tests are performed to validate the theoretical findings.

HYBRIDIZABLE DISCONTINUOUS GALERKIN METHOD FOR LINEAR HYPERBOLIC INTEGRO-DIFFERENTIAL EQUATIONS

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This article introduces hybridizable discontinuous Galerkin (HDG) approach to numerically approximate the solution of a linear hyperbolic integro-differential equation. A priori error estimates for semi-discrete and fully discrete schemes are developed. It is shown that optimal order of convergence is achieved for both scalar and flux variables. To achieve that, an intermediate projection is introduced for the semi-discrete error analysis, and it also shown that this projection achieves convergence of order $h^{k+3/2}$ for $k \geq 1$. Next, superconvergence is achieved for the scalar variable using element-by-element post-processing. For the fully discrete error analysis, the central difference scheme and the mid-point rule approximate the derivative and the integral term, respectively. Hence, the second order of convergence is achieved in the temporal direction. Finally, numerical experiments have been performed to validate the theory developed in this article.

A THREE STEPS TWO-GRID DISCONTINUOUS GALERKIN METHOD FOR THE OLDROYD MODEL OF ORDER ONE

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In this work, we propose and study a two-grid algorithm based on discontinuous Galerkin approximation for the equations of motion arising in Oldroyd model of order one. This algorithm is a combination of the following three steps. The first step consists of discretizing the nonlinear system utilizing discontinuous Galerkin method in the space direction and solving the system on a coarse grid $\mathcal{E}_H$ with grid size H. Then, in the second step, by employing the coarse grid solution, Newton's iteration type linearization is carried out and approximate solutions for the resulting system on a fine grid $\mathcal{E}_h$ with size h are calculated. A modified final solution is produced in the third step, which is a correction step for the solutions of the second step and is acquired by solving an appropriate linear problem on the fine grid. We further discretize the two-grid DG model in time, using the backward Euler method. Optimal fully discrete $\mathbf{L}^2$ and energy-norm error estimates for velocity and L^2-norm error estimates for pressure are derived for an appropriate choice of coarse and fine grid parameters. Finally, numerical results are presented to confirm the efficiency of the proposed scheme.

DIVERGENCE CONFORMING DISCONTINUOUS GALERKIN FINITE ELEMENT APPROXIMATION FOR DOUBLY DIFFUSIVE FLOWS

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It is the aim of this talk to discuss the H^{div}-conforming discontinuous Galerkin formulation for doubly diffusive flows. In addition, the efficiency and reliability of residual-based a posteriori error estimators for the steady, semi-discrete, and fully discrete problems are established. The resulting methods are applied to simulate the sedimentation of small particles in salinity-driven flows. Numerical results are presented that validate the theoretical estimates, and illustrate the effectiveness of the estimator in guiding adaptive solution algorithms.

This is joint work with R. Burger, P. Mandez and R. Ruiz-Baier.

References

[1] R. Bürger, A. Khan, P. E. Méndez, R. Ruiz-Baier Divergence-conforming methods for transient doubly-diffusive flows: A priori and a posteriori error analysis, IMA Journal on Numerical Analysis, 2023. https://doi.org/10.1093/imanum/drad090

CONVERGENCE OF ADAPTIVE FEMS FOR DISTRIBUTED ELLIPTIC OPTIMAL CONTROL PROBLEMS

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The adaptive finite element method is a powerful technique used to compute the numerical solution to differential equations with a minimal computational cost. In this talk, we will discuss the quasi-optimality of adaptive nonconforming finite element methods for distributed optimal control problems governed by m-harmonic operators for $m = 1, 2$. A variational discretization approach is employed and the state and adjoint variables are discretized using nonconforming finite elements. The general axiomatic framework that includes stability, reduction, discrete reliability, and quasi-orthogonality establishes the quasi-optimality. Numerical results demonstrate the theoretically predicted orders of convergence.

DOMAIN DECOMPOSITION METHODS FOR LINEARIZED AND NON-LINEAR CAHN-HILLIARD EQUATION

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The Cahn-Hilliard (CH) equation has emerged as a fundamental model in the study of phase separation phenomena in various physical and biological systems. Efficient and accurate numerical methods are essential for simulating these phenomena. We investigate the convergence properties of Domain Decomposition (DD) methods when applied to the CH equation at each discrete time step. By decomposing the computational domain into subdomains and employing suitable interface conditions, we aim to improve the convergence rates of various DD methods, specifically the parallel Schwarz method (PSM), the optimized Schwarz method (OSM), the Dirichlet-Neumann (DN) method, and the Neumann-Neumann (NN) method. Additionally, we study the numerical convergence behaviour of nonlinear variants of these methods. We also present the space-time DD methods for the CH equation, namely, the Schwarz Waveform Relaxation (SWR) and Dirichlet-Neumann Waveform Relaxation (DNWR) algorithms with appropriate numerical results.

ERROR BOUNDS FOR NUMERICAL APPROXIMATIONS OF FRACTIONAL HJB EQUATIONS

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We discuss monotone numerical methods for fractional and nonlocal HJB equations in strongly and weakly degenerate cases. These equations are dynamic programming equations of optimal control of SDEs driven by pure jump Levy processes. We discuss very precise (fractional) error bounds for diffusion corrected difference-quadrature methods. By precise fractional rate we mean, the convergence rates depend optimally on the order of the nonlocal operators. For weakly degenerate nonlocal equations, we obtain stronger regularity type estimates for both the viscosity solutions and numerical approximations.

WEAK GALERKIN DUAL MIXED FINITE ELEMENT METHOD FOR LINEAR PARABOLIC INTERFACE PROBLEMS

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In this talk, we present the weak Galerkin dual-mixed finite element method for linear parabolic interface problems in a bounded convex polygonal domain in $\mathbb{R}^2$. Both the spatially discrete and the fully discrete approximation based on the backward Euler approximations are analyzed. More precisely, we have derived a priori error bounds for both the solution and the flux variables, respectively in the $L^\infty(L^2)$ norm. The key feature of the error analysis is to define an appropriate elliptic projection operator combined with some Sobolev embedding theorems and some new approximation results for both the solution and the flux variables, respectively. Finally, some numerical experiments are performed to underline the theoretical findings.

WEAK GALERKIN FINITE ELEMENT METHODS FOR SECOND-ORDER WAVE EQUATIONS WITH POLYGONAL MESHES

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The wave equation is a primary archetype of a hyperbolic partial differential equations, and models propagation of various types of waves like elastic waves, sound waves in a gas or liquid, or electromagnetic waves. The key drivers of these applications include precise numerical methods so as to solve wave propagation problems. In the past few decades there has been remarkable progress in understanding and analyzing numerical algorithms for solving hyperbolic equations.

Recently, a novel finite element method, weak Galerkin finite element method (WG-FEM), was first developed by Wang and Ye for solving second order elliptic equations. In the WG-FEM procedure, unknowns are defined both inside the elements and on the element boundaries and the weak gradient of basis functions can be solved element by element locally. Comparing with traditional Galerkin FEMs, the WG methods are more appropriate for solving problems with discontinuous solutions or on complicated domains.

In this work, we describe weak Galerkin finite element methods for solving hyperbolic problems on polygonal meshes. We propose both semidiscrete and fully discrete schemes to numerically solve the second-order linear wave equation. For the time discretization, we have used implicit second order Newmark scheme. For suffciently smooth solutions, optimal order error estimate in the L^2 norm is shown to hold as $O(h^{k+1}+\tau^2)$, where h is the mesh size and τ the time step. An extensive set of numerical experiments are conducted to demonstrate the robustness, reliability, flexibility, and accuracy of the proposed method.

A LEAST-SQUARES-BASED WEAK GALERKIN FINITE ELEMENT METHOD FOR THE TIME-HARMONIC MAXWELLS EQUATIONS

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In this article, we propose and analyze a least-squares-based weak Galerkin finite element method (WG-FEM) for solving the indefinite time-harmonic Maxwells equations in $\mathbb{R}^d$ $(d = 2, 3)$. Superconvergence of order one for the discrete $\mathbf{H}^1$-like norm has been established. Numerical simulations show that the approximate solutions converge to the exact solutions with optimal rates in the L^2 norm on hybrid meshes. In addition, this method is shown to be an absolutely stable under low regularity requirements with high wave number.

AN HOC APPROACH ON NON-UNIFORM GRIDS FOR TIME-DEPENDENT ADVECTION-DIFFUSION EQUATIONS

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In this work, diffusion of a Gaussian pulse in two dimensions was revisited using a higher-order compact (HOC) finite difference scheme on nonuniform grids. We compared the results on non-uniform grids with those on uniform grids. Simulations on a suitable non-uniform grid were seen to be computationally more efficient than the uniform one in accurately capturing the phenomenon. Our numerical solutions were found to be in good agreement with the analytical solutions. This scheme was further implemented on time-dependent advection-diffusion equations including those governed by the Navier-Stokes equations.

ON FITTED MESH METHOD FOR SINGULARLY PERTURBED SEMILINEAR PARABOLIC PDES WITH NON-HOMOGENEOUS BOUNDARY DATA

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In the case of fully discrete numerical approximation of evolutionary PDEs, in particular, with non-homogeneous time-dependent boundary conditions; it is noticed that the classical evaluation of the boundary data usually causes the order reduction in time; and it becomes severe when the fractional-step method is used. In this work, we aim to develop and analyze an efficient higher-order numerical approximation for two-dimensional singularly perturbed semilinear parabolic convection-diffusion problems with time-dependent boundary data. To achieve the aim, we approximate the governing nonlinear problem by developing an efficient fractional-step fitted mesh method (FMM) followed by the extrapolation technique. The novelty of the current algorithm, other than its higher-order accuracy, is that the method can eliminate the order-reduction in time before and after the extrapolation with an appropriate evaluation of the boundary data; and at the same time, it can reduce the computational cost for solving the multi-dimensional problem by using the fractional-step method.

This is a joint work with Narendra Singh Yadav (SRM University AP, Andhra Pradesh)

CRANK-NICOLSON A POSTERIORI ERROR ESTIMATES FOR PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS WITH SMALL RANDOM INPUT DATA

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In this article, we present residual-based a posteriori error estimates for the parabolic partial differential equation (PDE) with small random input data. Such a class of PDEs arises due to a lack of complete understanding of the physical model. To this end, the perturbation technique [2019, Arch. Comput. Methods Eng., 26, pp. 1313-1377] is exploited to express the exact random solution in terms of the power series with respect to the uncertainty parameter, whence we obtain decoupled deterministic problems. Each problem is then discretized in space by the finite element method and advanced in time by the Crank-Nicolson scheme. Quadratic reconstructions are introduced to obtain optimal bounds in the temporal direction. The work generalizes the isotropic results for the deterministic parabolic PDEs obtained in [2009, SIAM J. Sci. Comput., 31, pp. 2757-2783] to the parabolic PDE with small random input data.

LAX-WENDROFF FLUX RECONSTRUCTION METHOD FOR HYPERBOLIC CONSERVATION LAWS

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The Lax-Wendroff method is a single step method for evolving time dependent solutions governed by partial differential equations, in contrast to Runge-Kutta methods that need multiple stages per time step. In this talk, we discuss a flux reconstruction version of the method in combination with a Jacobian-free Lax-Wendroff procedure that is applicable to general hyperbolic conservation laws. The method is of collocation type, is quadrature free and can be cast in terms of matrix and vector operations. Special attention is paid to the construction of numerical flux, including for non-linear problems, resulting in higher CFL numbers than existing methods, which is shown through Fourier analysis and yielding uniform performance at all orders. Numerical results up to fifth order of accuracy for linear and non-linear problems will be discussed to demonstrate the performance and accuracy of the method.